Answers to Coursebook questions – Chapter 5.3

1 The positive charge on the rod will attract electrons to sphere A, making sphere C positively charged.

So when they are separated, sphere A will be negative, B will be neutral and C positive.

2 **a** The distance of the centre of the square from each of the vertices is $a = \sqrt{0.050^2 + 0.050^2} = 0.0707\ \text{cm}$.

So the potential at the centre is $V = 4 \times \dfrac{kQ}{a} = 4 \times \dfrac{8.99 \times 10^9 \times 5 \times 10^{-6}}{0.0707} = 2.5 \times 10^6\ \text{V}$.

b The field at the centre is clearly zero – a positive charge placed there will be repelled equally from the four charges at the vertices, resulting in a net force of zero.

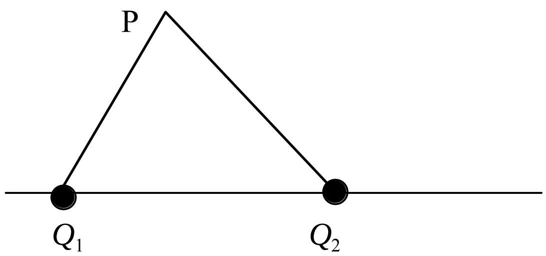
c The electric field is the gradient of the potential. The fact that the field is zero at the centre means that the potential near the centre is a stationary point.

3 **a** $V = 2 \times \dfrac{kQ}{d/2} = \dfrac{4kQ}{d}$

b

$$V = \frac{kQ}{d/2} - \frac{kQ}{d/2} = 0$$

4 A diagram is:



The potential at P is $V = \dfrac{8.99 \times 10^9 \times 2.0 \times 10^{-6}}{0.4} - \dfrac{8.99 \times 10^9 \times 4.0 \times 10^{-6}}{0.6} = -1.5 \times 10^4\ \text{V}$.

5 The work done is

$$W = q\Delta V = q\left(\frac{kQ}{r_2} - \frac{kQ}{r_2}\right) = 1.0 \times 10^{-3} \times \left(\frac{8.99 \times 10^9 \times 10}{2.0} - \frac{8.99 \times 10^9 \times 10}{10}\right) = 3.6 \times 10^7\ \text{J}.$$

6 The work done on the electron is

$$W = q\Delta V = q\left(\frac{kQ}{r} - 0\right) = (-1.6 \times 10^{-19}) \times \frac{8.99 \times 10^9 \times (-10)}{0.10} = +1.4 \times 10^{-7}\ \text{J}.$$

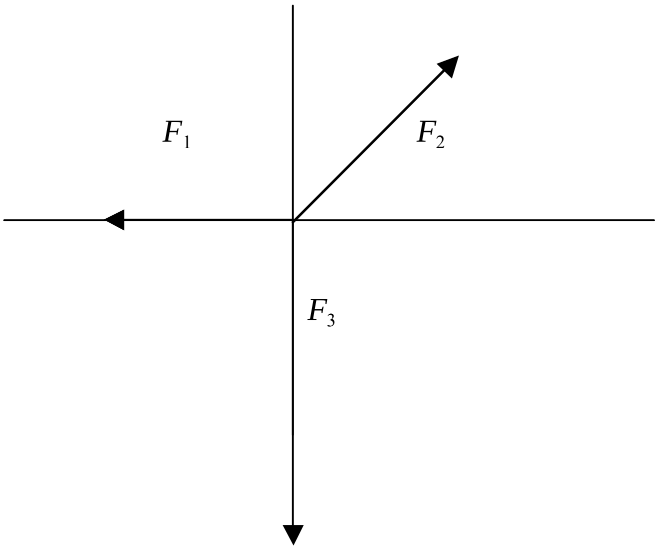
7 The work done ($W = q\Delta V$) is equal to the change in kinetic energy ($\frac{1}{2}mv^2$).

Hence

$$\frac{1}{2}\times 9.1\times 10^{-31}\times v^2 = 1.6\times 10^{-19}\times(200-100)$$

$$\Rightarrow v = \sqrt{\frac{2\times 1.6\times 10^{-19}\times 100}{9.1\times 10^{-31}}} = 5.9\times 10^{6}\ \text{m s}^{-1}.$$

8 **a** The forces are roughly as follows:



They have magnitudes:

$$F_1 = \frac{8.99\times 10^{9}\times 1\times 10^{-6}\times 2\times 10^{-6}}{0.05^2} = 7.19\ \text{N}$$

$$F_2 = \frac{8.99\times 10^{9}\times 4\times 10^{-6}\times 2\times 10^{-6}}{0.05^2 + 0.05^2} = 14.4\ \text{N}$$

$$F_3 = \frac{8.99\times 10^{9}\times 3\times 10^{-6}\times 2\times 10^{-6}}{0.05^2} = 21.6\ \text{N}$$

We must find the components of F_2 :

$F_{2x} = F_2 \cos 45° = 10.2 \text{ N}$ and $F_{2y} = F_2 \sin 45° = 10.2 \text{ N}$.

So the net force has components:

$F_x = 10.2 - 7.2 = 3.0 \text{ N}$ and $F_y = 10.2 - 21.6 = 11.4 \text{ N}$.

The net force is then $F = \sqrt{(11.4)^2 + (3.0)^2} = 11.8 \text{ N}$.

The direction of the net force is $\arctan\left(\frac{-11.4}{3.0}\right) = -75°$.

b The distance of the centre of the square from each of the vertices is $a = \sqrt{0.025^2 + 0.025^2} = 0.0354 \text{ cm}$. So the potential at the centre is

$$V = \frac{kQ_1}{a} + \frac{kQ_2}{a} + \frac{kQ_3}{a} + \frac{kQ_4}{a} = \frac{8.99\times 10^9}{0.0354} \times (-1\times 10^{-6} + 2\times 10^{-6} - 3\times 10^{-6} + 4\times 10^{-6})$$

$V = 5.1\times 10^5 \text{ V}$

c The work done is

$W = q\Delta V = q(V - 0) = 1.0 \times 10^{-9} \times 5.1 \times 10^5 = 5.1 \times 10^{-4} \text{ J}$.

9 a The dipole moment is the product of one of the charges in the dipole times their separation and so $a = \frac{6.2 \times 10^{-30}}{1.6 \times 10^{-19}} = 3.9 \times 10^{-11} \text{ m}$.

b Their equal and opposite forces on each of the charges of the dipole and so the net force is zero.

c The largest torque is when the dipole is normal to the field and equals

$\Gamma = F\frac{a}{2} + F\frac{a}{2} = Fa = qEa = 1.2 \times 10^{-23} \text{ N m}$.

10 **a** Charge will move until both spheres are at the same potential.

Then $\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$. By conservation of charge, $q_1 + q_2 = Q$, where Q is the charge on the one sphere originally.

Thus $\frac{q_1}{10} = \frac{q_2}{15} \Rightarrow 3q_1 = 2q_2$ and $q_1 + q_2 = 2.0$.

Hence $q_1 = \frac{2}{5} \times 2.0 = 0.80\ \mu\text{C}$ and $q_2 = \frac{3}{5} \times 2.0 = 1.2\ \mu\text{C}$.

b $\sigma_1 = \frac{0.80 \times 10^{-6}}{4\pi \times 0.10^2} = 6.4 \times 10^{-6}\ \text{C m}^{-2}$ and $\sigma_2 = \frac{1.2 \times 10^{-6}}{4\pi \times 0.15^2} = 4.2 \times 10^{-6}\ \text{C m}^{-2}$.

c $E_1 = \frac{kq_1}{r_1^2} = 4\pi k\sigma_1 = 4\pi \times 8.99 \times 10^9 \times 6.4 \times 10^{-6} = 7.2 \times 10^5\ \text{N C}^{-1}$ and

$E_2 = 4\pi k\sigma_2 = 4\pi \times 8.99 \times 10^9 \times 4.2 \times 10^5 = 4.8 \times 10^5\ \text{N C}^{-1}$.

d The electric field is largest for the sphere with the larger charge density. The wire has to be long so that the charge of one sphere will not affect the charge distribution on the other, so that both are uniformly charged.

11 You must draw lines that are normal to the equipotentials.

12 **a** The potential a distance x from the bottom plate is given by

$V = -250 + \frac{250 - (-250)}{0.15} x = (-250 + 3.33 \times 10^3 x)\ \text{V}$ and so at $x = 3.00\ \text{cm}$,

$V = (-250 + 3.33 \times 10^3 \times 0.0300) = -150\ \text{V}$. Therefore the electric potential energy of the charge is $E_P = qV = (-2.00 \times 10^{-6}) \times (-150) = 0.300\ \text{mJ}$.

b The potential at $x = 12.0$ cm is $V = (-250 + 3.33 \times 10^3 \times 0.120) = 150\ \text{V}$ and hence

$E_P = qV = (-2.00 \times 10^{-6}) \times 150 = -0.300\ \text{mJ}$.

c The work done must be

$W = q\Delta V = \Delta E_P = -0.300 - 0.300 = -0.600\ \text{J}$

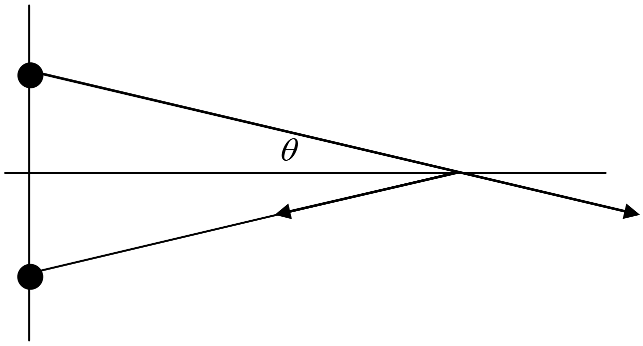
13 **a** The kinetic energy of the electron

$E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.59 \times 10^6)^2 = 1.15 \times 10^{-18}\ \text{J}$ gets converted to electric potential energy eV at the point where the electron stops. Hence the potential at P is $V = \frac{1.15 \times 10^{-18}}{1.6 \times 10^{-19}} = 7.19\ \text{V}$. Since the charge Q is negative the potential just found must be negative, i.e. $V = -7.19\ \text{V}$.

b

$$V = \frac{kQ}{r} \Rightarrow Q = \frac{Vr}{k} = \frac{(-7.19) \times 2.0 \times 10^{-10}}{9 \times 10^9} = -1.6 \times 10^{-19}\ \text{C}$$

14 **a** The field due to each of the charges has the direction shown. It is clear that the net field will point in the negative y – direction.

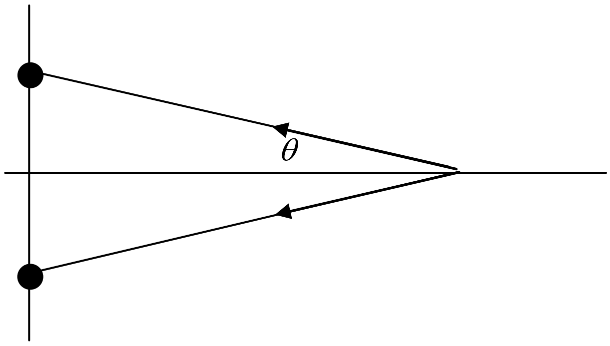


The magnitude of the field due to one of the charges is $E = \frac{kQ}{r^2} = \frac{kQ}{a^2 + d^2}$.

The y-component is $E_y = \frac{kQ}{a^2 + d^2}\sin\theta = \frac{kQ}{a^2 + d^2}\frac{a}{\sqrt{a^2 + d^2}} = \frac{kQa}{(a^2 + d^2)^{3/2}}$

and so the net field is $E_{net} = \frac{2kQa}{(a^2 + d^2)^{3/2}}$.

b For two negative charges:



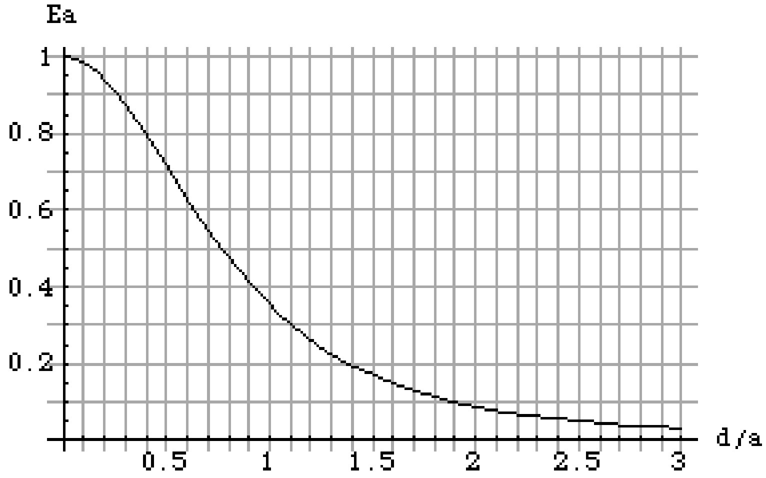
The net field is clearly directed to the left. It has magnitude

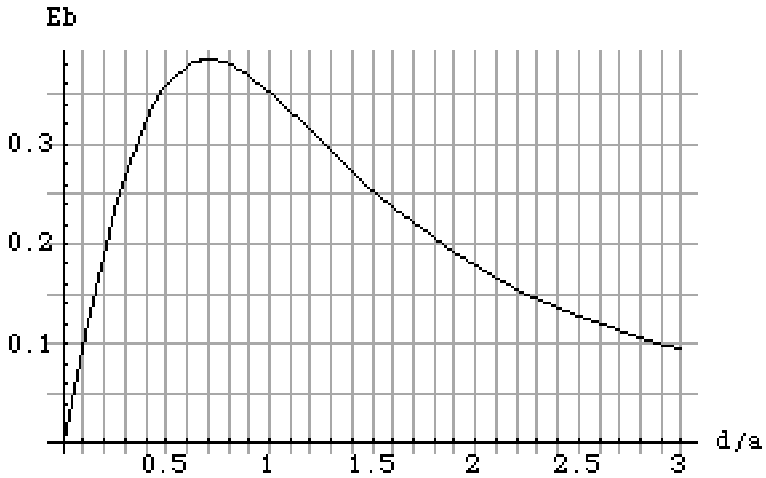
$E_{net} = 2E_x = \frac{kQ}{a^2 + d^2}\cos\theta = \frac{kQ}{a^2 + d^2}\frac{d}{\sqrt{a^2 + d^2}} = \frac{kQd}{(a^2 + d^2)^{3/2}}$.

c We have $E_a = \dfrac{kQa}{(a^2+d^2)^{3/2}} = \dfrac{kQ}{a^2}\dfrac{1}{\left(1+\dfrac{d^2}{a^2}\right)^{3/2}}$

and $E_b = \dfrac{kQd}{(a^2+d^2)^{3/2}} = \dfrac{kQ}{a^3}\dfrac{d}{\left(1+\dfrac{d^2}{a^2}\right)^{3/2}} = \dfrac{kQ}{a^2}\dfrac{d/a}{\left(1+\dfrac{d^2}{a^2}\right)^{3/2}}$

The plots are (the vertical axis is in units of $\dfrac{kQ}{a^2}$):





15 Since the field is the same as that of the charge Q and an equal and opposite image charge $-Q$ inside the plane the field at P is just $E = \dfrac{kQ}{d^2} - \dfrac{kQ}{9d^2} = \dfrac{8kQ}{9d^2}$ $\left(= \dfrac{8Q}{36\pi\varepsilon_0 d^2}\right)$ and is directed upwards.

16 **a** Radially in towards the centre.

b The net force on the charge is the electric force and so

$\frac{kq^2}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{kq^2}{mr}$.

c The total energy is the sum of the kinetic and the electric potential energies, i.e.

$$E_T = \frac{1}{2}mv^2 + \frac{k(-q)(q)}{r} = \frac{1}{2}\frac{kq^2}{r} - \frac{kq^2}{r} = -\frac{kq^2}{2r}.$$

d At the new orbit the total energy is greater: $E_T' = -\frac{kq^2}{4r}$.

Hence the energy required is $-\frac{kq^2}{4r} - \left(-\frac{kq^2}{2r}\right) = \frac{kq^2}{4r}$.

17 The initial potential energy of the three protons is zero. When at the vertices of the triangle of side a the potential energy is $E_P = 3 \times \frac{k(\mathrm{e})(\mathrm{e})}{a} = \frac{3ke^2}{a}$ since there are three pairs of charges a distance a apart.

This evaluates to $E_P = \frac{3 \times 8.99 \times 10^9 \times (1.6 \times 10^{-19})^2}{5.0 \times 10^{-16}} = 1.4 \times 10^{-12}\ \mathrm{J} \approx 8.6\ \mathrm{MeV}$.

This is the energy that must be supplied.

